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Effect of vertical heterogeneity on the onset of ferroconvection in a Brinkman porous medium

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Isothermal/insulated boundaries

Abstract The onset of ferroconvection in a Brinkman porous medium is studied for different forms of vertical heterogeneity permeability function $\Gamma(z)$. The eigenvalue problem is solved numerically using the Galerkin method for isothermal/insulated rigid-ferromagnetic boundary conditions. It is observed that the onset of ferromagnetic convection can be either hastened or delayed depending on the type of heterogeneity of the porous medium as well as thermal boundary conditions. The effect of increasing magnetic number is to hasten the onset of ferroconvection for all choices of $\Gamma(z)$. Although the measure of nonlinearity of magnetization is to hasten the onset of ferroconvection in the case of isothermal boundaries, it shows no influence on the onset criterion when the boundaries are insulated. Further, the deviation in critical Darcy–Rayleigh number between different forms of $\Gamma(z)$ is found to be not so significant with an increase in the Darcy number.

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1. Introduction

Ferrofluids (magnetic fluids) are commercially manufactured colloidal liquids usually formed by suspending mono-domain nanoparticles (their diameter is typically 10 nm) of magnetite in non-conducting liquids such as heptanes, kerosene, water, etc., and they are also called magnetic nanofluids. These fluids are magnetized in the presence of an external magnetic field, and due to their both liquid and magnetic properties, they have emerged as reliable materials capable of solving complex engineering problems. An authoritative introduction to this fascinating subject along with their applications is provided [1–3]. The magnetization of ferrofluids depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. This leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is similar to buoyancy driven convection. The theory of ferroconvection instability in a ferrofluid layer began

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Nomenclature

A	ratio of heat capacities	$\vec{q}=(u,v,w)$	velocity vector
a	horizontal wave number,	p	pressure
$D = d/dz$	differential operator	ρ_f	fluid density
$K(z)$	variable permeability of the porous medium	μ_f	fluid viscosity
$\vec{M}$	magnetization of the ferrofluid	$\tilde{\mu}_f$	effective viscosity
$\vec{H}$	magnetic field	μ_0	magnetic permeability
T	temperature	ρ_0	reference density
$Da=\tilde{\mu}_f K_0/\mu_f d^2$	Modified Darcy number	ε	porosity of the porous medium
$M_1=\mu_0 K_p^2\beta/(1+\chi)\alpha_t\rho_0 g$	Magnetic number	κ	thermal diffusivity
$M_3 = (1 + M_0/H_0)/(1 + \chi)$	Measure of nonlinearity of magnetization	α_t	thermal expansion coefficient
$R_D=\alpha_t g\beta K_0 d^2/\nu\kappa$	Darcy–Rayleigh number	$\beta = T_L - T_U/d$	temperature gradient
$\Gamma(z)=K_0/K(z)$	Permeability heterogeneity function	∇^2	Laplacian operator
K_0	Mean value of $K(z)$	*Subscripts*	
$\chi=(\partial M/\partial H)_{H_0,T_a}$	magnetic susceptibility,	f	fluid
$K_p=-(\partial M/\partial T_f)_{H_0,T_a}$	pyromagnetic co-efficient	b	basic state
$T_a = (T_L + T_U)/2$	average temperature of the fluid		

with Finlayson [4] and extensively continued over the years [5–10].

Thermal convection of ferrofluids saturating a porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment of chemicals, and emplacement of geophysically imageable liquids into particular zones for subsequent imaging, etc. The stability of the magnetic fluid penetration through a porous medium in a high uniform magnetic field oblique to the interface is studied [11]. Thermal convection of ferrofluid saturating a porous medium in the presence of a vertical magnetic field is studied by several authors [12–18]. Recently, Nanjundappa et al. [19] investigated the onset of buoyancy and surface tension forces in a magnetized ferrofluid saturated horizontal Brinkman porous layer with temperature dependent viscosity.

The effect of heterogeneity in either permeability or thermal conductivity or both on thermal convective instability in a layer of porous medium is of importance since there can occur dramatic effects in the case of heterogeneity (Braester and Vadasz [20], Nield and Bejan [21] and references therein). The porous domain being heterogeneous is common in many engineering applications due to series of horizontal layers in each of which the permeability is uniform. Besides, flow of ferrofluids through porous media was motivated by the potential use of ferrofluids to stabilize fingering in oil recovery processes. In such situations the presence of heterogeneities is common and it may affect the flow of ferrofluids through porous media. Under the circumstances, investigating the influence of heterogeneity of permeability on ferromagnetic convection in a layer of porous medium is of practical interest and also warranted. However, it appears that majority of the studies on ferromagnetic convection in a porous medium deal with the case of homogeneous porous medium, and the case of heterogeneity has been largely neglected. In view of this, Shivakumara et al. [22] studied the onset of ferromagnetic convection in a heterogeneous Darcy porous medium using a local thermal non-equilibrium model.

Nonetheless, there is ample scope for further investigations on ferromagnetic convection in a heterogeneous porous medium. The intent of the present study is to emphasize various forms of vertical heterogeneity (property variation in the vertical direction) in the permeability of the porous medium on the onset of ferromagnetic convection in a magnetized ferrofluid-saturated horizontal layer of Brinkman porous medium. The resulting eigenvalue problem is solved numerically using the Galerkin method for isothermal/insulated rigid-ferromagnetic boundaries.

2. Mathematical formulation

We consider an incompressible magnetized ferrofluid-saturated horizontal layer of Brinkman heterogeneous porous medium of characteristic thickness d in the presence of a uniform applied magnetic field in the vertical direction. The lower surface is held at constant temperature T_L, while the upper surface is at $T_U\ (<T_L)$. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and the z-axis directed vertically upward in the presence of gravitational field $\vec{g}$. The Boussinesq approximation on the density is made. The governing basic equations for thermal convection in a ferrofluid-saturated heterogeneous porous medium consist of the balances of mass, linear momentum, energy and they are respectively given by the following equation: [1,21]

$$\nabla\cdot\vec{q}=0. \tag{1}$$

$$\rho_0\left[\frac{1}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{1}{\varepsilon^2}(\vec{q}.\nabla)\vec{q}\right]=-\nabla \mathrm{p}+\rho_0\{1-\alpha_t(T-T_a)\}\vec{g}-\frac{\mu_f}{K(z)}\vec{q}+\tilde{\mu}_f\nabla^2\vec{q}+\mu_0(\vec{M}\cdot\nabla)\vec{H} \tag{2}$$

$$A\frac{\partial T}{\partial t}+(\vec{q}\cdot\nabla)T=\kappa\nabla^2 T \tag{3}$$

where $\vec{q} = (u, v, w)$ is the velocity vector, p the pressure, $\vec{M}$ the magnetization, $\vec{H}$ the magnetic field intensity, T the temperature, μ_f the dynamic viscosity, $\tilde{\mu}_f$ the effective viscosity, ρ_0 the reference density, α_t the thermal expansion coefficient, μ_0 the magnetic permeability of vacuum, μ_f the permeability of the porous medium, ε the porosity of the porous medium, $K(z)$ the variable permeability of the porous medium, κ the thermal diffusivity, $T_a = (T_L + T_U)/2$ the average temperature and A the ratio of heat capacities.

The Maxwell equations in the magnetostatic limit are as follows:

$$\nabla \cdot \vec{B} = 0 \tag{4}$$

$$\nabla \times \vec{H} = 0 \quad \text{or} \quad \vec{H} = \nabla\varphi \tag{5}$$

where, $\vec{B}$ is the magnetic induction and φ is the magnetic potential.

Further, $\vec{B}$, $\vec{M}$ and $\vec{H}$ are related by

$$\vec{B} = \mu_0(\vec{M} + \vec{H}). \tag{6}$$

It is assumed that the magnetization is aligned with the magnetic field, but allowed a dependence on the magnitude of the magnetic field as well as temperature (Finlayson [3]) and thus

$$\vec{M} = M(H, T)\frac{\vec{H}}{H} \tag{7}$$

where, $M = |\vec{M}|$ and $H = |\vec{H}|$. The magnetic equation of state, following Finlayson [3], is taken as

$$M = M_0 + \chi(H - H_0) - K_p(T - T_a) \tag{8}$$

where $M_0 = M(H_0, T_a)$ is the saturation magnetization, $\chi = (\partial M/\partial H)_{H_0,T_a}$ is the magnetic susceptibility, $K_p = -(\partial M/\partial T)_{H_0,T_a}$ is the pyromagnetic co-efficient. The basic state is quiescent and the solution is

$$p_b(z) = p_0 - \rho_0 g z - \frac{1}{2}\rho_0\alpha_t g\beta z(z-d) - \frac{\mu_0 M_0 K_p \beta}{1+\chi}z - \frac{\mu_0 K_p^2\beta^2}{2(1+\chi)^2}z(z-d)$$

$$T_b(z) = T_a - \beta(z - d/2)$$

$$\vec{H}_b(z) = \left[H_0 - \frac{K_p\beta}{1+\chi}\left(z - \frac{d}{2}\right)\right]\hat{k}$$

$$\vec{M}_b(z) = \left[M_0 + \frac{K_p\beta}{1+\chi}\left(z - \frac{d}{2}\right)\right]\hat{k} \tag{9}$$

where, $\beta = \Delta T/d = (T_L - T_U)/d$ is the temperature gradient, $\hat{k}$ is the unit vector in the z-direction and the subscript b denotes the basic state. We superimpose perturbations on the basic solution as $\vec{q} = \vec{q}'$, $p = p_b + p'$, $T = T_b + T'$, $\vec{H} = \vec{H}_b + \vec{H}'$ and $\vec{M} = \vec{M}_b + \vec{M}'$. Following the standard linear stability analysis procedure, performing the normal mode analysis and non-dimensionalizing the variables in the form

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), t^* = \frac{\kappa}{d^2}t, W^* = \frac{d}{\kappa}W, \Theta^* = \frac{1}{\beta d}\Theta, \Phi^* = \frac{(1+\chi)}{K_p\beta d^2}\Phi \tag{10}$$

the non-dimensional governing equations (after dropping the asterisks for simplicity and noting the principle of exchange of stability holds) can then be shown to be

$$\left[Da(D^2 - a^2) - \Gamma(z)\right](D^2 - a^2)W - D\Gamma(z)DW = -a^2 R_D[M_1 D\Phi - (1 + M_1)\Theta] \tag{11}$$

$$(D^2 - a^2)\Theta = -W \tag{12}$$

$$(D^2 - a^2 M_3)\Phi - D\Theta = 0 \tag{13}$$

Here, $D = d/dz$ is the differential operator, $a = \sqrt{\ell^2 + m^2}$ the overall horizontal wave number, W the amplitude of vertical component of velocity, Θ the amplitude of temperature, Φ the amplitude of magnetic potential, $Da = \tilde{\mu}_f K_0/\mu_f d^2$ the modified Darcy number, $R_D = \alpha_t g\beta d^4/\nu\kappa$ the Darcy–Rayleigh number, $M_1 = \mu_0 K_p^2\beta/(1+\chi)\alpha_t\rho_0 g$ the magnetic number, $M_3 = (1 + M_o/H_o)/(1 + \chi)$ the measure of nonlinearity of magnetization, $\Gamma(z) = K_0/K(z)$ the permeability heterogeneity function and K_0 the mean value of $K(z)$.

The function $\Gamma(z)$ is chosen in the following form:

$$\Gamma(z) = 1 + \delta_1(z - 1/2) + \delta_2(z^2 - 1/3) \tag{14}$$

where δ_1 and δ_2 are constants and it may be noted that the above quadratic function has a unit mean. For the homogeneous porous medium case, $\delta_1 = 0 = \delta_2$.

Eqs. (11)–(13) are to be solved subject to appropriate boundary conditions.

The boundaries are considered to be rigid, ferromagnetic and either isothermal or insulated to temperature perturbations.

The boundary conditions are

$$W = DW = \Theta \quad \text{or} \quad D\Theta = \Phi = 0 \text{ at } z = 0, 1. \tag{15}$$

3. Numerical solution

Eqs. (11)–(13) together with the boundary conditions given by (15) constitute an eigenvalue problem with R_D as the eigenvalue. The resulting eigenvalue problem is solved numerically using the Galerkin technique. Accordingly, $W(z)$, $\Theta(z)$ and $\Phi(z)$ are expanded in the series form

$$W = \sum_{i=1}^{n} A_i W_i(z), \ \Theta(z) = \sum_{i=1}^{n} B_i\Theta_i(z), \ \Phi(z) = \sum_{i=1}^{n} C_i\Phi_i(z) \tag{16}$$

where A_i, B_i and C_i are unknown coefficients. Multiplying momentum Eq. (11) by $W_j(z)$, energy Eq. (12) by $\Theta_j(z)$, and magnetization Eq. (13) by $\Phi_j(z)$; performing the integration by parts with respect to z between $z = 0$ and 1, and using the boundary conditions, we obtain the following system of linear homogeneous algebraic equations:

$$C_{ji}A_i + D_{ji}B_i + E_{ji}C_i = 0 \tag{17}$$

$$F_{ji}A_i + G_{ji}B_i = 0 \tag{18}$$

$$H_{ji}B_i + I_{ji}C_i = 0 \tag{19}$$

The coefficients $C_{ji} - I_{ji}$ involve the inner products of the base functions and are given by

$$C_{ji} = Da\left[\langle D^2W_jD^2W_i\rangle + 2a^2\langle DW_jDW_i\rangle + a^4\langle W_jW_i\rangle\right] + \langle\left(1 + \delta_1(z - 1/2) + \delta_2(z^2 - 1/3)\right)DW_jDW_i\rangle - \langle(\delta_1 + 2\delta_2 z)W_jDW_i\rangle + a^2\langle\left(1 + \delta_1(z - 1/2) + \delta_2(z^2 - 1/3)\right)W_jW_i\rangle$$

$$D_{ji} = a^2 R_D(1 + M_1)\langle W_j\Theta_i\rangle,$$

$$E_{ji} = -a^2 R_D M_1 \langle W_j D\Phi_i \rangle,$$

$$F_{ji} = -\langle \Theta_j W_i \rangle,$$

$$G_{ji} = \langle D\Theta_j D\Theta_i \rangle + a^2 \langle \Theta_j \Theta_i \rangle$$

$$H_{ji} = \langle \Phi_j D\Theta_i \rangle,$$

$$I_{ji} = \langle D\Phi_j D\Phi_i \rangle + a^2 M_3 \langle \Phi_j \Phi_i \rangle \tag{20}$$

where the inner product is defined as $\langle \cdots \rangle = \int_0^1 (\cdots) dz$.

The base functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ are assumed in the following form:

$$W_i = (z^4 - 2z^3 + z^2) T^*_{i-1}, \quad \Phi_i = (z^3 - 3z^2 + 2z) T^*_{i-1}$$

$$\Theta_i = (z^2 - z) T^*_{i-1} (\text{isothermal}),$$
$$\Theta_i = z^2 (2z - 3) T^*_{i-1} (\text{insulated}) \tag{21}$$

where, T^*_is ($i \in N$) are the modified Chebyshev polynomials, such that $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ satisfy the corresponding boundary conditions. The characteristic equation formed from (17)–(19) for the existence of non-trivial solution is solved numerically for different values of physical parameters as well as for different forms of $\Gamma(z)$. The Newton–Raphson method is used to obtain the Darcy–Rayleigh number RD as a function of wave number a when all the parameters and functions are fixed and the bisection method is built in to locate the critical stability parameters (R_{Dc}, a_c) to the desired degree of accuracy. It is observed that the results are converged by taking six terms in the Galerkin expansion.

4. Results and discussion

The onset of ferromagnetic convection in a layer of heterogeneous Brinkman porous medium heated from below has been investigated for two types of temperature boundary conditions, namely rigid-isothermal and rigid-insulated to temperature perturbations. The eigenvalue problem is solved numerically using the Galerkin method for four different models of vertical heterogeneity permeability function $\Gamma(z)$ viz.

(i) $F1$: $\Gamma(z) = 1$ (homogeneous),
(ii) $F2$: $\Gamma(z) = 1 + (z - 1/2)$ (linear variation in z),
(iii) $F3$: $\Gamma(z) = 1 + (z^2 - 1/3)$ (only quadratic variation in z) and
(iv) $F4$: $\Gamma(z) = 1 + (z - 1/2) + (z^2 - 1/3)$ (general quadratic variation in z).

To know the accuracy of the method employed, it is instructive to look at the critical Darcy–Rayleigh number R_{Dc} and the corresponding wave number a_c at various levels of the Galerkin approximation. The numerically computed values of R_{Dc} and the corresponding a_c for $M_1 = M_3 = 1$ and $Da = 0.001$, 0.01 for various forms of $\Gamma(z)$ are shown in Table 1 for isothermal boundaries. From the tabulated values it is clear that the results converge for six terms in the Galerkin expansion. Further inspection of the table reveals that R_{Dc} turns out to be same for permeability heterogeneity functions of models $F1$ and $F2$ as well as models $F3$ and $F4$, and also the permeability heterogeneous function of model $F4$ is more stable compared to model $F1$ if single term is considered in the Galerkin expansion. To the contrary, values of R_{Dc} for different vertical heterogeneity of permeability functions differ and more importantly model $F3$ shows more stabilizing effect than model $F4$ at higher order Galerkin method. This suggests the limitation of single term Galerkin method in analyzing the problem considered.

The results obtained for isothermal boundaries are shown in Figs. 1–5, while Fig. 6 exhibits the results for insulated boundaries. Figs. 1–3 exhibit neutral curves in the (R_D, a)-plane for two values of M_1(=0.1, 0.5 with $M_3 = 1$ and $Da = 0.001$), M_3(=1, 5 with $Da = 0.001$ and $M_1 = 1$) and Da(=0.001, 0.01 with $M_3 = 1$ and $M_1 = 1$), respectively for different forms of $\Gamma(z)$. The neutral stability curves exhibit single but different minimum with respect to the wave number for various forms of $\Gamma(z)$ and their shape is identical in the form to that of the Brinkman–Benard problem. For each of the forms of $\Gamma(z)$, the effect of increasing M_1 (see Fig. 1) and M_3 (see Fig. 2) is to decrease the region of stability, while opposite is the trend with increasing Da (see Fig. 3).

The sensitivity of various forms of vertical heterogeneity of permeability function $\Gamma(z)$ on the stability characteristics of the system is made clear in Figs. 4 and 5. Figs. 4 and 5(a) illustrate the variation of critical Darcy–Rayleigh number R_{Dc} as a function of M_1 for two values of M_3(=1, 10 with $Da = 0.001$) and Da(=0.001, 0.01 with $M_3 = 1$), respectively. The size of M_1 is related to the importance of magnetic force as compared to gravitational force. It is observed that the effect of increasing M_1 leads to decrease in the Rayleigh number suggesting that the ferrofluids carry heat more efficiently than the ordinary vis-

Table 1 Comparison of critical Darcy–Rayleigh and the corresponding wave numbers for different orders of approximations in the Galerkin expansion for $M_1 = M_3 = 1$.

Approximations									
Da	Models	$i = j = 1$		$i = j = 2$		$i = j = 5$		$i = j = 6$	
		R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c	R_{Dc}	a_c
0.001	F1	23.728	3.305	25.813	3.361	23.594	3.277	23.594	3.277
	F2	23.728	3.305	25.580	3.382	22.971	3.294	22.975	3.295
	F3	24.037	3.418	25.875	3.502	23.761	3.434	23.764	3.434
	F4	24.037	3.418	25.005	3.584	21.795	3.515	21.802	3.515
0.01	F1	31.657	3.243	34.463	3.291	33.276	3.292	33.276	3.292
	F2	31.657	3.243	34.350	3.297	33.035	3.298	33.033	3.298
	F3	32.006	3.317	34.701	3.374	33.565	3.378	33.564	3.378
	F4	32.006	3.317	34.262	3.397	32.824	3.399	32.819	3.399

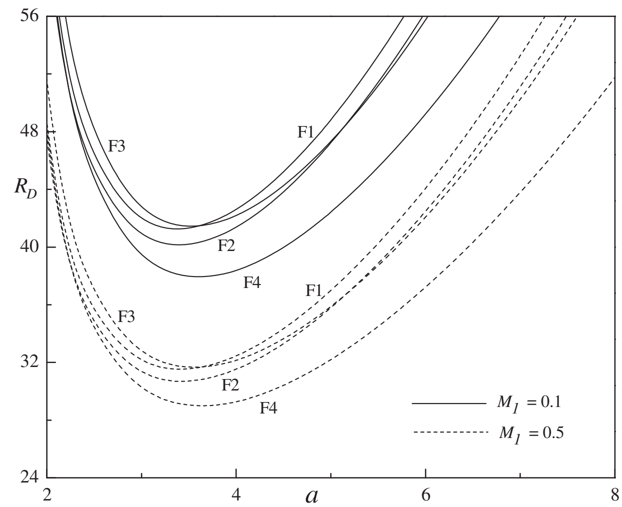

Figure 1 Neutral curves for different values of M_1 when $M_3 = 1$ and $Da = 0.001$ for different forms of $\Gamma(z)$.

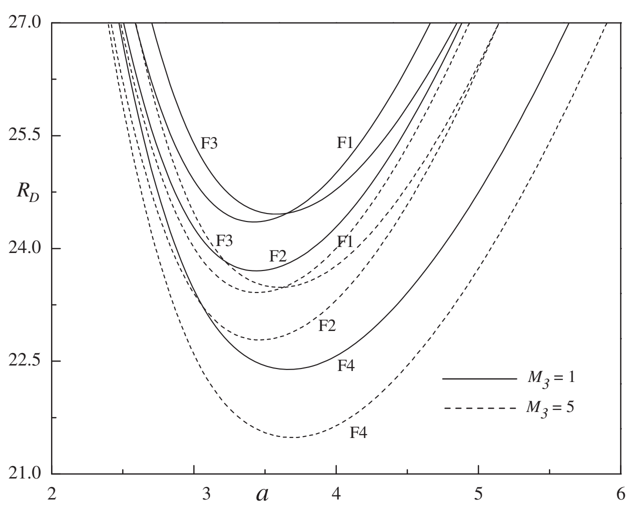

Figure 2 Neutral curves for different values of M_3 when $M_1 = 1$ and $Da = 0.001$ for different forms of $\Gamma(z)$.

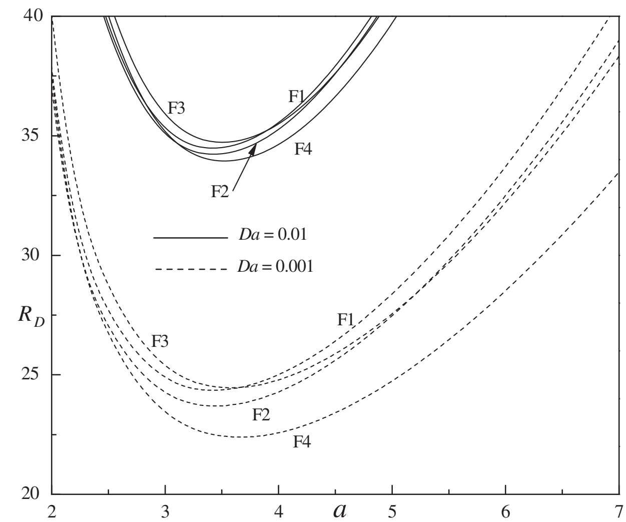

Figure 3 Neutral curves for different values of Da when $M_3 = 1$ and $M_1 = 1$ for different forms of $\Gamma(z)$.

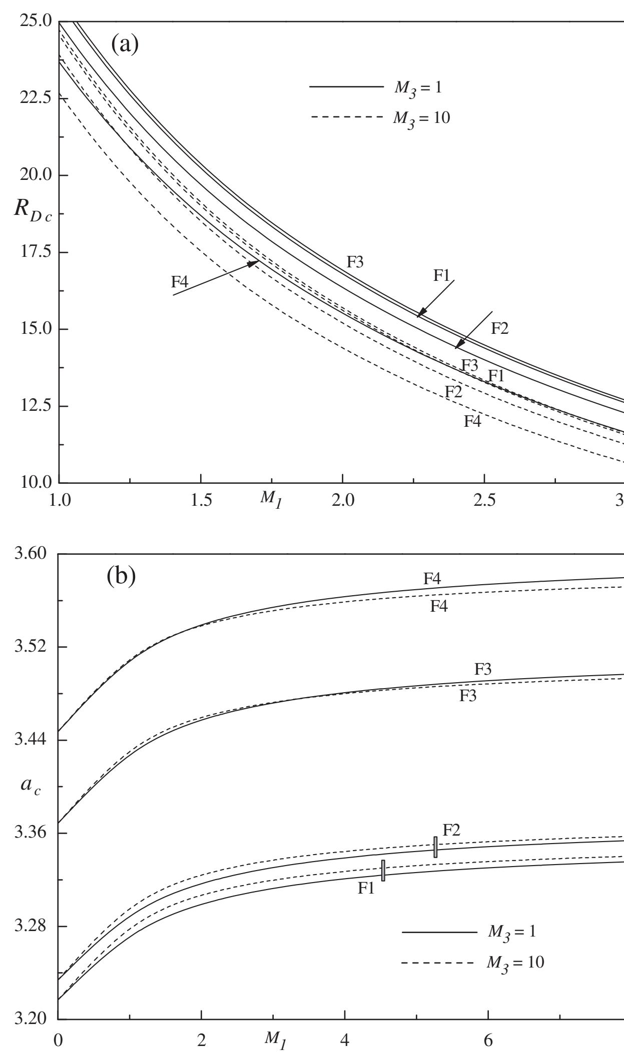

Figure 4 Variation of (a) R_{Dc} (b) a_c for different values of M_3 when $Da = 0.001$ for different forms of $\Gamma(z)$.

cous fluids. This is due to an increase in the destabilizing magnetic force with increasing M_1, which favors the fluid to flow more easily. The case $M_1 = 0$ corresponds to convective instability in an ordinary viscous fluid saturating a porous medium. Similar is the case with increasing M_3(Fig. 4a). This is because, a higher value of M_3 would arise either due to a larger pyromagnetic coefficient or larger temperature gradient. Both these factors are conducive for generating a larger gradient in the Kelvin body force field, possibly promoting the instability. Further inspection of the figures reveals that the system is more stable if the form of $\Gamma(z)$ is of the model $F3$ and then least stable for models $F1$ and $F2$. Thus the onset of ferromagnetic convection is either hastened or delayed depending on the type of heterogeneity in the permeability of the porous medium. In addition, the deviation in the critical Darcy–Rayleigh numbers between different forms of permeability heterogeneity function $\Gamma(z)$ is found to be not so significant. Fig. 5(a) illustrates that increase in the value of Da (equivalently increase in the ratio of

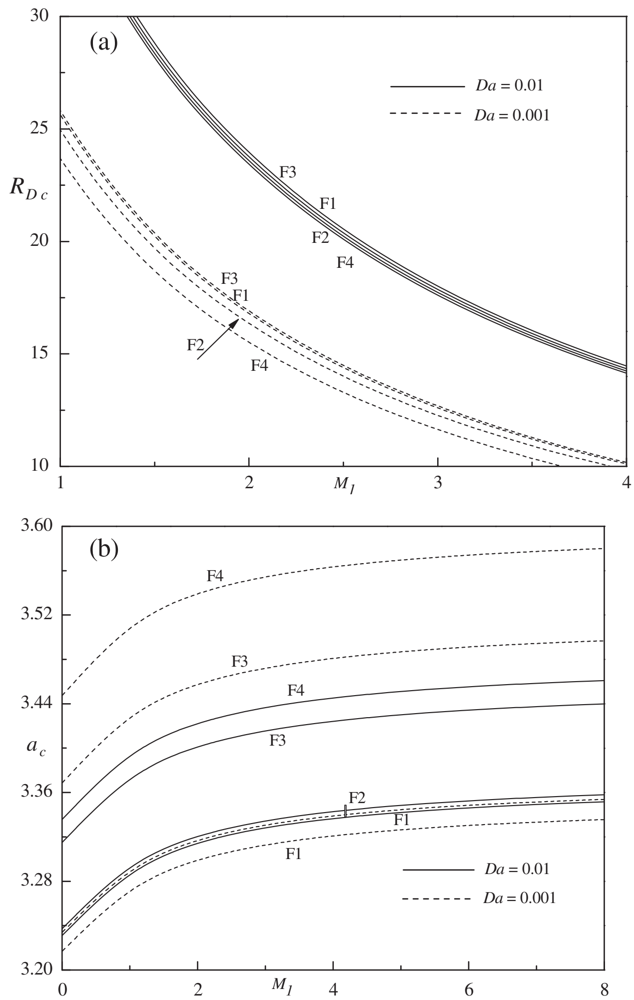

Figure 5 Variation of (a) R_{Dc} and (b) a_c for different values of Da when $M_3 = 1$ for different forms of $\Gamma(z)$.

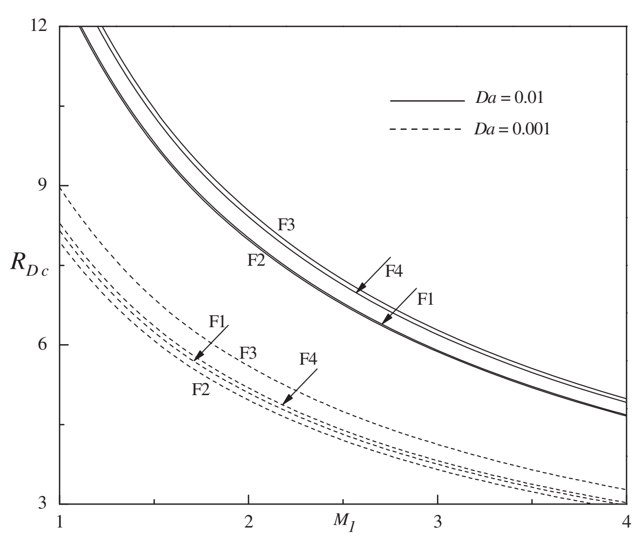

Figure 6 Variation of R_{Dc} for different values of Da when $M_3 = 1$ for different forms of $\Gamma(z)$.

viscosities, $\tilde{\mu}_f/\mu_f$) is to increase the critical Rayleigh number and hence its effect is to delay the onset of ferromagnetic convection due to an increase in viscous diffusion.

Figs. 4 and 5(b) illustrate the variation of critical wave number a_c, as a function of M_1 for various forms of vertical heterogeneity of permeability function $\Gamma(z)$, for two values of $M_3(=1,\ 10$ with $Da = 0.001)$ and $Da(=0.001,\ 0.01$ with $M_3 = 1)$, respectively. The critical wave number a_c increases with increasing $M1$. Thus the effect of increasing M_1 is to contract the size of convection cells. Further, increase in Da and M_3 is to increase a_c for the models $F1$ and $F2$, while opposite is the trend for the models $F3$ and $F4$. An inspection of the figures also reveals that the critical wave number is higher for model $F4$ followed by $F3$, then $F2$ and the least for $F1$.

The critical wave number is found to be vanishingly small and also M_3 has no effect on the onset in the case of insulated boundaries. The critical Darcy–Rayleigh number obtained as a function of M_1 for various forms of $\Gamma(z)$ is illustrated in Fig. 6 for two values of $Da = 0.001$ and 0.01. From the figure it is observed that the system is more stable if the form of $\Gamma(z)$ is of the model $F3$ and least stable for model $F2$. These phenomena are different from those observed in the case of isothermal boundaries. Besides, insulated boundaries are found to be more destabilizing when compared to isothermal boundaries. This may be attributed to the fact that all the energy is available to the system in the case of insulated boundaries which facilitates the system to become unstable at lower values of critical Darcy–Rayleigh number.

5. Conclusions

The principal results of the foregoing study may be summarized as follows:

(i) Irrespective of different forms of vertical heterogeneity permeability function $\Gamma(z)$, the onset of ferromagnetic convection retains its unimodal shape with one distinct minimum which defines the critical Darcy–Rayleigh number and the critical wave number for various values of physical parameters.
(ii) The system is more stable when $\Gamma(z)$ varies quadratically (model $F3$) and least stable if it has the form of general quadratic variation (model $F4$) with depth z in the case of isothermal boundaries, whereas, models $F3$ and $F2$ are found to be more stable and unstable, respectively in the case of insulated boundaries. Thus the onset of ferromagnetic convection can be either hastened or delayed depending on the type of heterogeneity of the porous medium as well as thermal boundary conditions.
(iii) Increase in the value of magnetic number M_1 and the measure of nonlinearity of magnetization M_3 is to hasten the onset of ferromagnetic convection, while increasing the Darcy number Da shows stabilizing effect on the system due to increase in viscous diffusion. In the case of insulated boundaries M_3 has no effect on the stability characteristics of the system.
(iv) Increasing M_1 is to increase the critical wave number. Also, increase in Da and M_3 is to increase critical wave number a_c for the models $F1$ and $F2$, while opposite is the trend for the models $F3$ and $F4$. Compared to the homogeneous porous medium case, the critical wave

number is higher if the permeability is heterogeneous and more so if $\Gamma(z)$ has the form of general quadratic variation (model $F4$). The critical wave number is vanishingly small when the boundaries are insulated to temperature perturbations.

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